



K26P 1305

Reg. No. :

Name :

Second Semester M.Sc. Degree (C.B.S.S. – Supplementary)
Examination, April 2026
(2022 Admission)
MATHEMATICS
MAT2C07 : Measure and Integration

Time : 3 Hours

Max. Marks : 80

PART – A

Answer **any four** questions from this Part. **Each** question carries **4** marks. **(4×4=16)**

1. When can you say that a set E is Lebesgue measurable ? Show that if $m^*(E) = 0$ then E is measurable.
2. Show that the outer measure is translation invariant.
3. Prove that if f and g are measurable. $|f| \leq |g|$ a. e., and g is integrable, then f is integrable.
4. Let f and g be integrable functions. If $f \leq g$ a. e., then prove that $\int f \, dx \leq \int g \, dx$.
5. Given $f = g$ a. e. (μ), where μ is a complete measure. Prove that if f is measurable, so is g .
6. Define $\|f\|_\infty$. Prove that $\|f + g\|_\infty \leq \|f\|_\infty + \|g\|_\infty$.

PART – B

Answer **any four** questions from this Part without omitting any Unit. **Each** question carries **16** marks. **(4×16=64)**

Unit – I

7. a) State and prove Lebesgue's Monotone Convergence Theorem.
b) Show that Lebesgue outer measure of an interval equals its length.

P.T.O.



8. a) Let E be a measurable set. Then for every y the set $E + y = [x + y : x \in E]$ is measurable and the measures are the same.
- b) Let $\{E_i\}$ be a sequence of measurable sets. Show that
- i) if $E_1 \subseteq E_2 \subseteq \dots$, then $m(\lim E_i) = \lim m(E_i)$
 - ii) if $E_1 \supseteq E_2 \supseteq \dots$ and $m(E_i) < \infty$ for each i , then $m(\lim E_i) = \lim m(E_i)$
9. a) Show that $\int_1^\infty \frac{dx}{x} = \infty$.
- b) If f be a non-negative measurable function then show that there exists a sequence $\{\phi_n\}$ of measurable simple functions such that, for each x , $\phi_n(x) \uparrow f(x)$.

Unit – II

10. a) If μ is a measure on a ring R and if the function μ^* is defined on $H(R)$ by
- $$\mu^*(E) = \inf \left[\sum_{n=1}^{\infty} \mu(E_n) : E_n \in R, n = 1, 2, \dots, E \subseteq \bigcup_{n=1}^{\infty} E_n \right],$$
- then prove that
- i) for $E \in R$, $\mu^*(E) = \mu(E)$
 - ii) μ^* is an outer measure on $H(R)$.
- b) Show that $\lim_{\beta \rightarrow \infty} \int_a^b f(x) \sin \beta x dx = 0$.
11. a) If f is Riemann integrable and bounded over the finite interval $[a, b]$, then prove that f is integrable and $R \int_a^b f dx = \int_a^b f dx$.
- b) Let μ^* be the outer measure on $H(R)$ defined by μ on R , then prove that S^* contains $S(R)$, the σ -ring generated by R .
12. a) If μ is a σ -finite measure on a ring R , then prove that it has a unique extension to the σ -ring $S(R)$.
- b) Show that $\lim_{n \rightarrow \infty} \int_a^\infty \frac{n^2 x e^{-n^2 x^2}}{1 + x^2} dx = 0$, for $a > 0$, but not for $a = 0$.



Unit – III

13. Let $1 < p < \infty, 1 < q < \infty$ $\frac{1}{p} + \frac{1}{q} = 1$ and let $f \in L^p(\mu)$, $g \in L^q(\mu)$, then prove that $fg \in L^1(\mu)$ and $\int |fg| d\mu \leq \left(\int |f|^p d\mu \right)^{\frac{1}{p}} \cdot \left(\int |g|^q d\mu \right)^{\frac{1}{q}}$.
- When does the equality occur ? Justify your answer.
14. a) If $f \in L^1(\mu)$ and $g \in L^\infty(\mu)$ then prove that $fg \in L^1(\mu)$ and $\|fg\|_1 \leq \|f\|_1 \|g\|_\infty$.
- b) If $1 \leq p < \infty$ and $\{f_n\}$ is a sequence in $L^p(\mu)$ such that $\|f_n - f_m\|_p \rightarrow 0$ as $n, m \rightarrow \infty$, then prove that there exist a function f and a subsequence $\{n_i\}$ such that $\lim_{n_i} f_{n_i} = f$ a. e. Also prove that $f \in L^p(\mu)$ and $\lim_{n_i} \|f_{n_i} - f\|_p = 0$.
15. State and prove Minkoski's Inequality. When does the equality occur ? Justify.

